Journal of Sound and Vibration (1998) **211**(5), 729–734

IDENTIFICATION OF TRANSVERSE CRACK LOCATION IN FLEXURAL VIBRATIONS OF FREE–FREE BEAMS

M. Boltezar, B. Strancar and A. Kuhelj

University of Ljubljana, Faculty of Mechanical Engineering, Aškerčeva 6, 1000 *Ljubljana, Slovenia*

(*Received* 26 *June* 1996, *and in final form* 18 *April* 1997)

This paper deals with the crack identification procedure for free–free uniform beams in flexural vibrations. The model of a transverse crack includes an equivalent linear spring, connecting two segments of a beam. By measuring the changes of natural frequencies in flexural vibrations it is possible to study the inverse problem—the crack site identification. The method is based on the assumption that the crack stiffness does not depend on the frequency of vibration. It requires at least two natural frequencies to be measured which are changed due to the crack existence. The comparison with the crack sites, identified by measuring both axial and flexural vibrations, showed better results for the flexural vibration case.

1. INTRODUCTION

In recent years, the dynamics of cracked structural members, especially beams, has been the subject of much research. A crack in a structural member can be thought of as a local flexibility, and as such, depends on crack depth. The existence of a crack further reduces the natural frequencies of the structure, so consequently, by measuring changes in natural frequencies the location of crack can be identified.

The problem of dynamic behavior of cracked beams has been studied by several authors. Adams *et al*. [1] were among the first to use the equivalent spring to model a damaged section. They used axial vibrations of one-dimensional beams to locate the damaged region in inverse problem. Rizos *et al*. [2] proposed measurements of amplitudes at two distinct points for a cantilever beam in flexural vibration in order to locate the depth and magnitude of a crack. Narkis [3] studied the dynamics of a cracked, simply supported beam for either bending or axial vibrations. He showed that the only information required for accurate crack identification is the variation of the first two natural frequencies due to a crack.

A short recent review, together with some other references can be found in Boltezar *et al*. [4]. A considerable experimental work is described in Montalvao E Silva *et al*. [5].

The approach developed by Adams *et al*. [1] has been adopted for axial vibrations and further extended to flexural vibrations. The crack is modelled as a torsional spring, following Rizos *et al*. [2]. The solution is then applied to the inverse problem in order to locate the damage site from measuring natural frequencies of flexural vibrations of a cracked beam.

0022–460X/98/150729 + 06 $25.00/0/sv971410

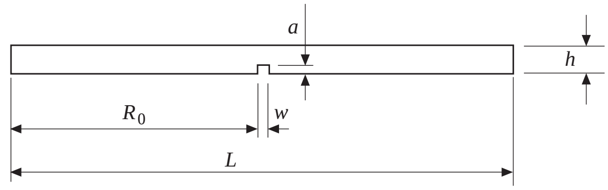

Figure 1. Schematic model of the cracked free–free beam.

2. THEORY

The physical model being investigated in this work is a uniform Euler–Bernoulli beam, supported free–free as shown in Figure 1. The length of a beam is L, and R_0 denotes the true position of a transverse surface crack extending uniformly across the width of the beam, starting at the left end. The beam has constant cross-section A and area moment of inertia I. Young's modulus E and mass density ρ are also constant. The effect of material damping on natural frequencies is assumed to be negligible. Starting with the Euler–Bernoulli equation of motion governing the free flexural vibration of a uniform beam gives

$$EI\,\partial^4 y/\partial x^4 + \rho A\,\partial^2 y/\partial t^2 = 0. \tag{1}$$

In the vicinity of the crack, this equation does not hold due to the abrupt change of the cross-section. Since only flexural vibrations are considered, the rotational crack compliance is assumed to be dominant in the local flexibility matrix. This is the basis of the decision to model a crack with a torsional massless and dimensionless spring.

A mode of harmonic vibration can be written as

$$V(x) = A_1 \sin \lambda x + A_2 \cos \lambda x + A_3 \sinh \lambda x + A_4 \cosh \lambda x, \tag{2}$$

where A_i are constants to be determined from the boundary conditions.

The problem of overcoming the published values of the material properties is solved by calculating the effective value of Young's modulus, following Adams *et al*. [1]. It can be calculated as

$$E_{eff} = \rho A \omega^2/\lambda^4 I, \tag{3}$$

where the dimensions of the beam are measured, the value of mass density is taken from the literature, values for λ_i are computed according to the boundary conditions and the ω are measured natural frequencies. Effective values for Young's modulus must be found for each natural vibrating mode of the uncracked beam separately.

Now consider the cracked beam from Figure 1. It is modelled by two uniform segments of beams on both sides of the crack and the torsional spring between which model the crack itself and is shown in Figure 2 [2]. Mode shapes can be written for both segments

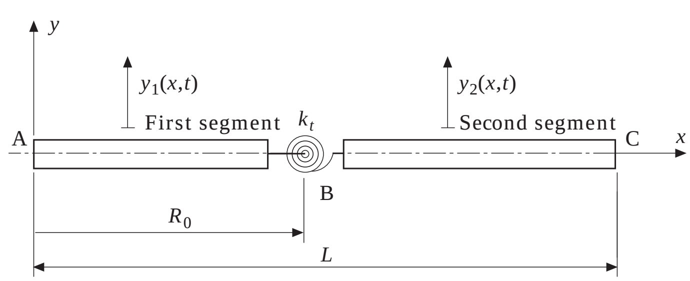

Figure 2. Model of the cracked beam with linear spring.

using equation (2). Boundary conditions include zero moments and shear forces at both ends A and C. The continuity condition at the crack position B requires equality of displacements, moments and shear forces at both sides of a crack.

$$y_{1B} = y_{2B}, \qquad y''_{1B} = y''_{2B}, \qquad y'''_{1B} = y'''_{2B}. \tag{4}$$

The angular displacement between two beam segments at point B is related to the moment of this section and to crack flexibility $1/k_t$

$$y'_{1B} + M_1/k_t = y'_{2B}. \tag{5}$$

Boundary and continuity conditions result in a set of 8 homogeneous linear algebraic equations for eight unknown coefficients. For a non-trivial solution, the determinant defined in equation (6) must be zero (see page 4). In the determinant of equation (6) $\delta = k_t/EI$ denotes relative torsional stiffness that models the crack.

In the procedure of identifying the crack location, first the natural frequencies of flexural vibrations of the free–free beam are measured. Next, the eigenvalues λ_i are computed using the effective values of the Young's modulus, which were determined for the uncracked beam. Finally, the value $\delta = k_t/EI$ is computed from equation (6) as a function of possible crack location R for each of the natural frequencies separately and shown for all of the frequencies in the same graph. Based on the assumption that the crack stiffness does not depend on the frequency of vibration, the values of crack stiffness must be the same at the crack position for all of the measured natural frequencies. So one is looking for intersection of the curves $\delta = k_t/EI$ to determine the crack location. This idea was originally shown by Adams *et al*. [1] for axial vibrations of one-dimensional bars and is here extended to flexural vibrations of one-dimensional beams.

3. EXPERIMENT

The experimental investigation was done on a homogeneous straight steel beam with rectangular cross-sectional area $h \times b = 15 \times 30{\cdot}2$ mm. The length L was 498·5 mm and the true relative crack position R_0/L was 0·38.

The damage was artificially introduced by a saw, so consequently the width of a cut was 1 mm and remained open during the testing. The depth of the crack has been varied. The boundary conditions were free–free; the beam was supported by a thin nylon rope the length of which was 1·5 m. The excitation was impulsive. Vibration signals were acquired by two accelerometers, both mounted at one end of the beam. One of them was attached to pick up axial and the second to pick up flexural vibrations.

4. RESULTS

The positions of resonance peaks were experimentally measured for the first 6 natural frequencies of flexural vibrations for the undamaged beam. They were 306·78, 841·19, 1635·25, 1672·0, 3933·0 and 5397·5 Hz. Relative frequency resolution was about 0·01% according to the used zoom technique.

From the measured natural frequencies of the undamaged beam the values of the effective Young's modulus were computed using equation (3). They were found to be 191 900 MPa for the first and 189 880 MPa for the second natural frequency. By computing them it is possible to overcome the influence of the mass of both accelerometers at one end of the beam. The mass of each was 13 g and consequently the first few natural frequencies decreased by 1·3–1·4% for flexural vibration.

$$
f(R,\lambda,\delta)=\left|\begin{array}{cccc:cccc}
0 & -1 & 0 & 1 & 0 & 0 & 0 & 0\\
-1 & 0 & 1 & 0 & 0 & 0 & 0 & 0\\
\hdashline
-\sin\lambda R & -\cos\lambda R & \sinh\lambda R & \cosh\lambda R & \sin\lambda R & \cos\lambda R & -\sinh\lambda R & -\cosh\lambda R\\
-\cos\lambda R & \sin\lambda R & \cosh\lambda R & \sinh\lambda R & \cos\lambda R & -\sin\lambda R & -\cosh\lambda R & -\sinh\lambda R\\
\cos\lambda R-\frac{\lambda}{\delta}\sin\lambda R & -\sin\lambda R-\frac{\lambda}{\delta}\cos\lambda R & \cosh\lambda R+\frac{\lambda}{\delta}\sinh\lambda R & \sinh\lambda R+\frac{\lambda}{\delta}\cosh\lambda R & -\cos\lambda R & \sin\lambda R & -\cosh\lambda R & -\sinh\lambda R\\
\sin\lambda R & \cos\lambda R & \sinh\lambda R & \cosh\lambda R & -\sin\lambda R & -\cos\lambda R & -\sinh\lambda R & -\cosh\lambda R\\
\hdashline
0 & 0 & 0 & 0 & -\sin\lambda L & -\cos\lambda L & \sinh\lambda L & \cosh\lambda L\\
0 & 0 & 0 & 0 & -\cos\lambda L & \sin\lambda L & \cosh\lambda L & \sinh\lambda L
\end{array}\right|=0 \qquad (6)
$$

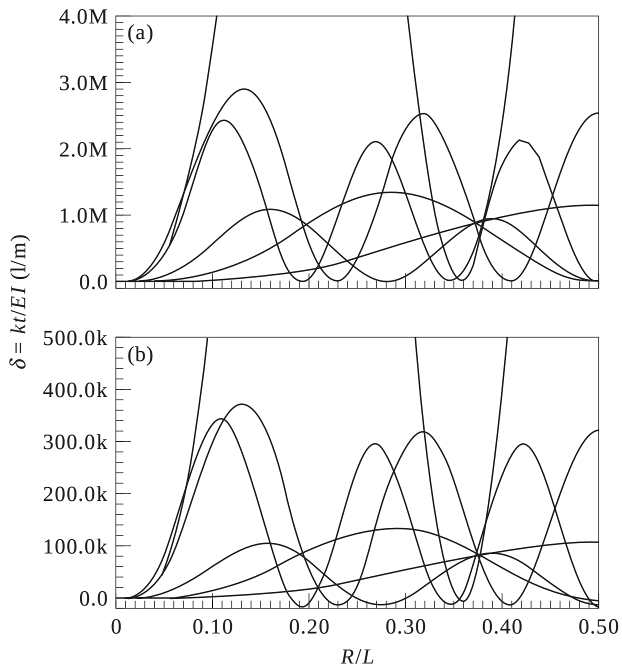

Figure 3. Relative crack stiffness versus crack position for the first 6 natural frequencies and for 2 different relative crack depths: (a) $a/h = 11{\cdot}3\%$; (b) $a/h = 36{\cdot}3\%$.

The changes of the first 6 natural frequencies were then successively measured for each increasing depth of a saw cut. The relative crack stiffness was finally computed using equation (6) for several different values of relative crack depth. The results are plotted against the relative possible crack position R/L and are shown in Figure 3 for two different values of the relative crack depth, 11·3 and 36·3%.

The true crack position was at $R_0/L = 0{\cdot}38$. In order to compare the adopted approach the natural frequencies of axial vibrations were also measured and the crack location identified, directly repeating the procedure of Adams *et al*. [1]. In Table 1 the differences of the identified crack location versus true values are shown both for axial and for flexural

Table 1

Identification of the crack location for axial and flexural vibrations, difference to true crack site

Relative crack depth (%)	Axial vibrations		Flexural vibrations	
	R/L	Difference (%)	R/L	Difference (%)
4	0·37	−1·1	0·38	−0·1
8	0·35	−3·1	0·379	−0·2
11·3	0·34	−4·1	0·38	−0·1
15	0·33	−5·1	0·38	−0·1
21·3	0·35	−3·1	0·376	−0·5
27·7	0·36	−2·1	0·377	−0·4
36·63	0·368	−1·3	0·376	−0·5
51	0·37	−1·1	0·374	−0·7

vibrations for the beam under consideration. As can be seen from Table 1, the error of the damage site location for flexural vibrations is less than 1% in the range from 4–51% of the relative crack depth. For the tested beam, one found in the present case of free–free support the use of flexural vibrations to be superior over axial.

5. CONCLUSIONS

A method for identifying the open transverse crack location of a uniform beam, supported free–free was developed by extending the known approach from axial to flexural vibrations. The crack was theoretically simulated by an equivalent linear spring. In order to determine crack location, the changed natural frequencies of flexural vibrations must be measured with great accuracy. To overcome the influence of unsatisfactory precision of published data for material properties such as Young's modulus as well as the effect of the accelerometers' mass to natural frequencies, the known principle of the effective value of the Young's modulus was adopted, resulting from the measurements of the natural frequencies of uncracked beam for each vibrating mode.

The method is based on the assumption that the stiffness of the equivalent spring which models the crack must be the same for all of the vibrating modes of flexural natural vibrations. So consequently, by plotting the relative stiffness along the length of the beam for distinct natural frequencies, the crack site can be identified by the intersection of the curves. Six natural frequencies were measured in order to improve the statistics of the curves' intersection.

The method gives reliable and accurate results (within 1%) at relative crack depths bigger than 5%. By also measuring the natural frequencies of axial vibrations, the comparison between flexural and axial based identification procedures showed better results with the former.

REFERENCES

1. R. D. Adams, P. Cawley, C. J. Pye and B. J. Stone 1978 *Journal of Mechanical Engineering Science* **20**, 93–100. A vibration technique for non-destructively assessing the integrity of structures.
2. P. F. Rizos, N. Aspragathos and A. D. Dimarogonas 1990 *Journal of Sound and Vibration* **138**, 381–388. Identification of crack location and magnitude in a cantilever beam from the vibration modes.
3. Y. Narkis 1994 *Journal of Sound and Vibration* **172**, 549–558. Identification of crack location in vibrating simply supported beams.
4. M. Boltezar, B. Strancar and A. Kuhelj 1997 *Sixth International Conference on Recent Advances in Structural Dynamics*, *ISVR*, *Southampton*, 14–17 July. **1**, 671–679. Assessing the transverse crack location in flexural vibrations of straight uniform beams.
5. J. M. Montalvao e Silva and A. J. M. Araujo Gomes 1990 *Experimental Mechanics* **30**, 20–25. Experimental dynamic analysis of cracked free–free beams.